DYNAMO MODELING OF STELLAR ACTIVITY CYCLES

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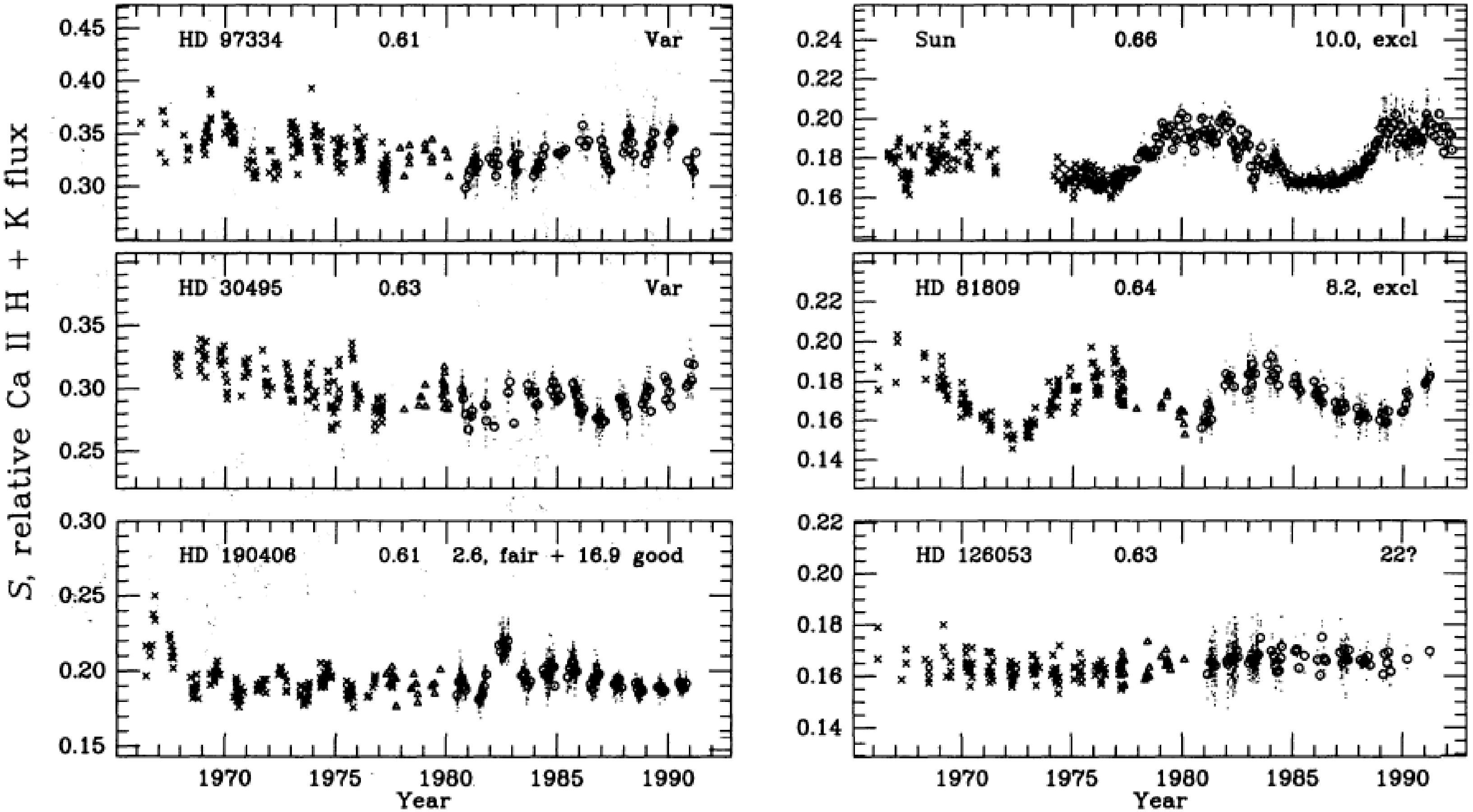
Space Climate 5

H-K PROJECT

- Observations of stellar cycles in the Ca II HK band since the mid-1960s



Baliunas et al. (1995)

METHOD

- Goal: to numerically reproduce observationally inferred relationships relating rotation period, cycle period, luminosity, Rossby number, etc.
- 20 simulations with various rotation rates (5) & timescales for thermal forcing (4)
- EULAG-HD: global 3D hydrodynamical simulation
- $\alpha\Omega$ mean-field dynamo model using EULAG's output to specify large-scale flows and α-tensor

Mean-Field Dynamo

- Scale separation into a "mean" part and a "fluctuating" part
- Azimuthal averaging $\langle\cdot\rangle$ of the fluctuating component gives zero
- $\boldsymbol{\varepsilon} = \langle \boldsymbol{u}' \times \boldsymbol{B}' \rangle, \quad \varepsilon_i = \alpha_{ij}\langle B_j \rangle + \beta_{ijk}\partial_k \langle B \rangle_j + \cdots$
- Assume that the large-scale fields are axisymmetric, then

$$\langle \boldsymbol{B} \rangle = \langle B_\phi \rangle \hat{\mathrm{e}}_\phi + \nabla \times \left(\langle A_\phi \rangle \hat{\mathrm{e}}_\phi \right)$$

- Turbulent EMF becomes a source term in the mean-field dynamo equations

MEAN-FIELD DYNAMO

- $\alpha\Omega$-dynamo equations neglecting meridional circulation:

$$\frac{\partial A}{\partial t} = \eta\left(\nabla^2 - \frac{1}{\varpi^2}\right)A + C_\alpha \alpha B$$

$$\frac{\partial B}{\partial t} = \eta\left(\nabla^2 - \frac{1}{\varpi^2}\right)B + \frac{1}{\varpi}\left(\frac{d\eta}{dr}\right)\frac{\partial(\varpi B)}{\partial r} + C_\Omega \varpi\left[\nabla \times \left(A\hat{\mathrm{e}}_\phi\right)\right]\cdot(\nabla\Omega)$$

- $\varpi = r\sin\theta$, θ polar angle
- $C_\alpha = \frac{\alpha_0 R}{\eta_0}$ and $C_\Omega = \frac{\Omega_0 R^2}{\eta_0}$
- $C_\alpha = 12.5,\ C_\Omega = \frac{1.5 D_{\mathrm{crit}}}{C_\alpha},\ \Delta\eta = 10$

RELATING α TO KINETIC HELICITY

- For an isotropic and homogeneous turbulent flow, α is diagonal and can be expressed as follow (Second Order Correlation Approximation):

$$\alpha(r,\theta) = -\frac{\tau(r)}{3} h_v(r,\theta),$$

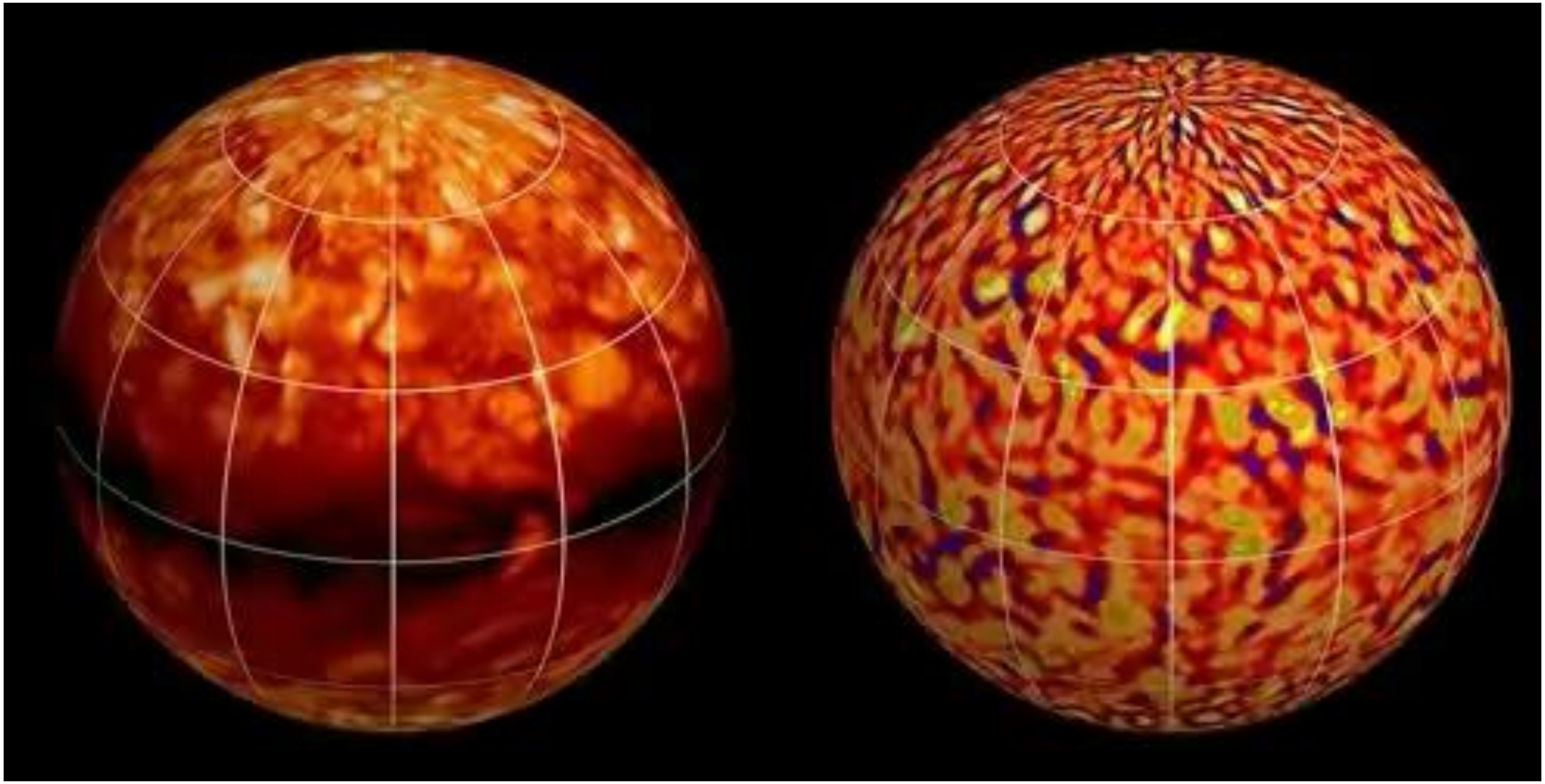
where $h_v = \langle \boldsymbol{u}' \cdot \nabla \times \boldsymbol{u}' \rangle$ and $\tau = \frac{H_\rho}{u'_{\mathrm{rms}}}$

- Last piece missing, amplitude limiting non-linearity

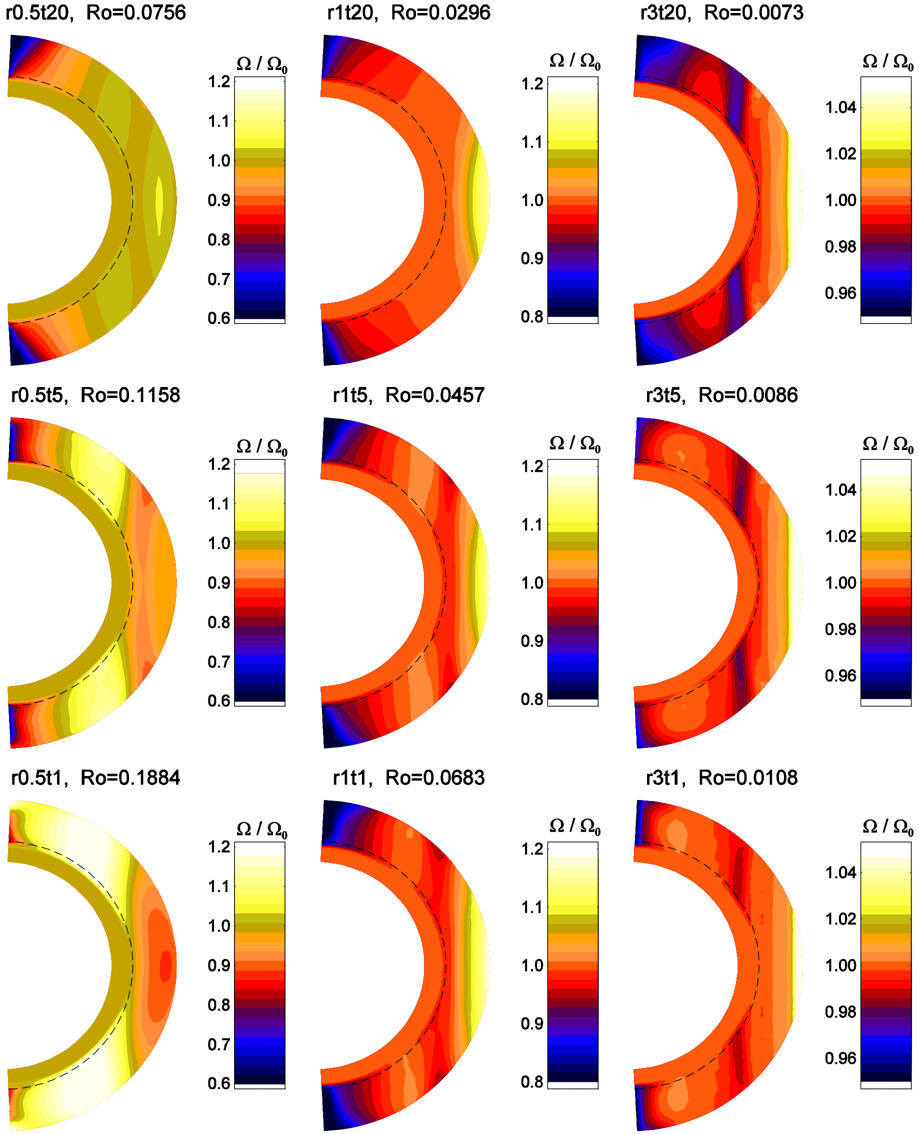
$$\alpha \longrightarrow \frac{\alpha}{1 + \left(B/B_{\mathrm{eq}}\right)^2}$$

EULAG-HD RESULTS

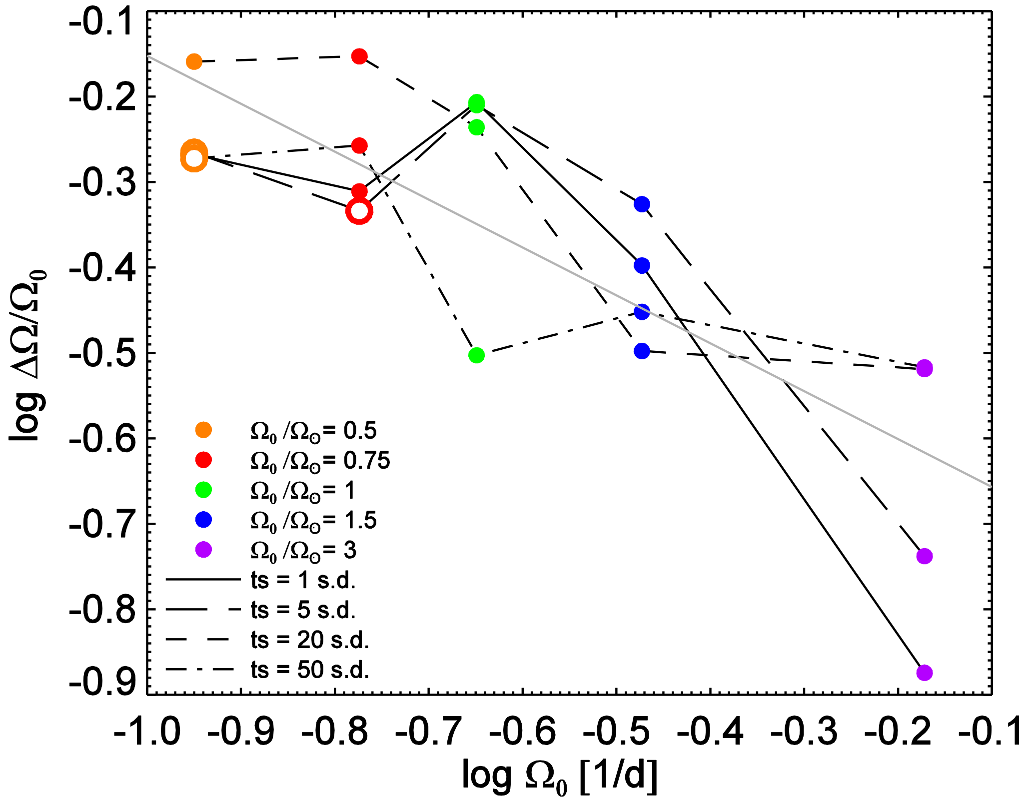
Temperature | Radial component of the velocity



EULAG-HD RESULTS

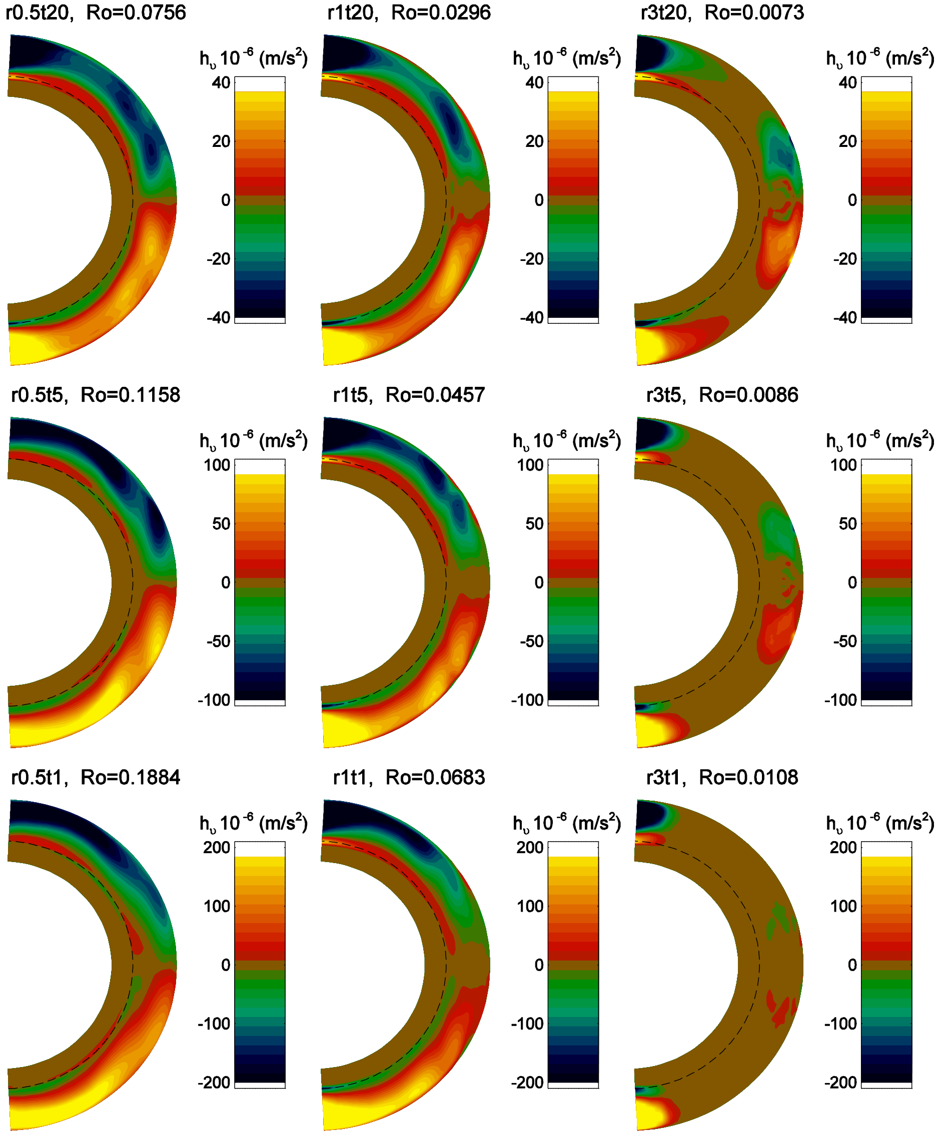


VARIATION OF ΔΩ



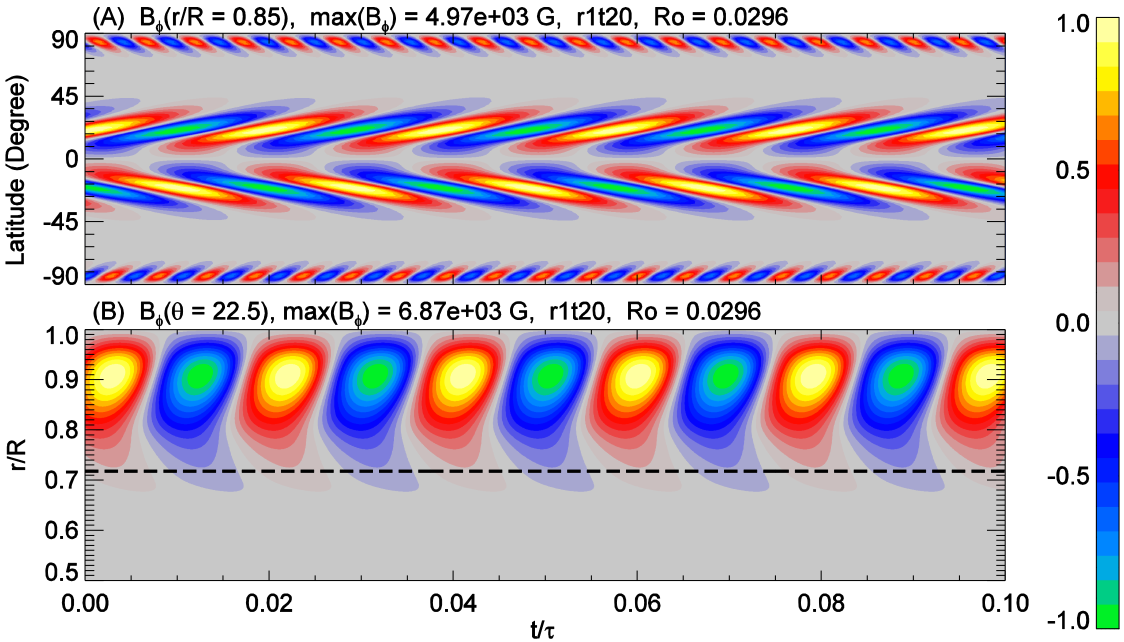
- $\Delta\Omega/\Omega_0 \propto {\Omega_0}^{-0.56}$ with all points, or $\Delta\Omega/\Omega_0 \propto {\Omega_0}^{-0.69}$ omitting simulations without equatorial acceleration
- Brown et al. (2008) found $\Delta\Omega \propto {\Omega_0}^{0.3}$

EULAG-HD RESULTS

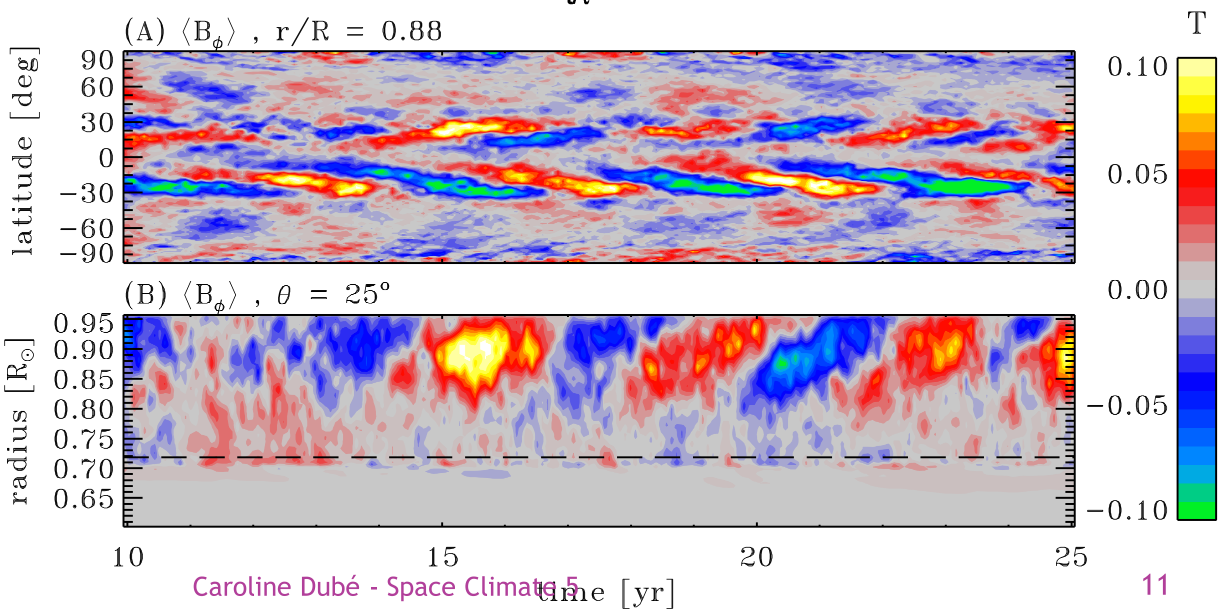


DYNAMO SOLUTIONS

$\alpha\Omega$ dynamo model



3D global MHD



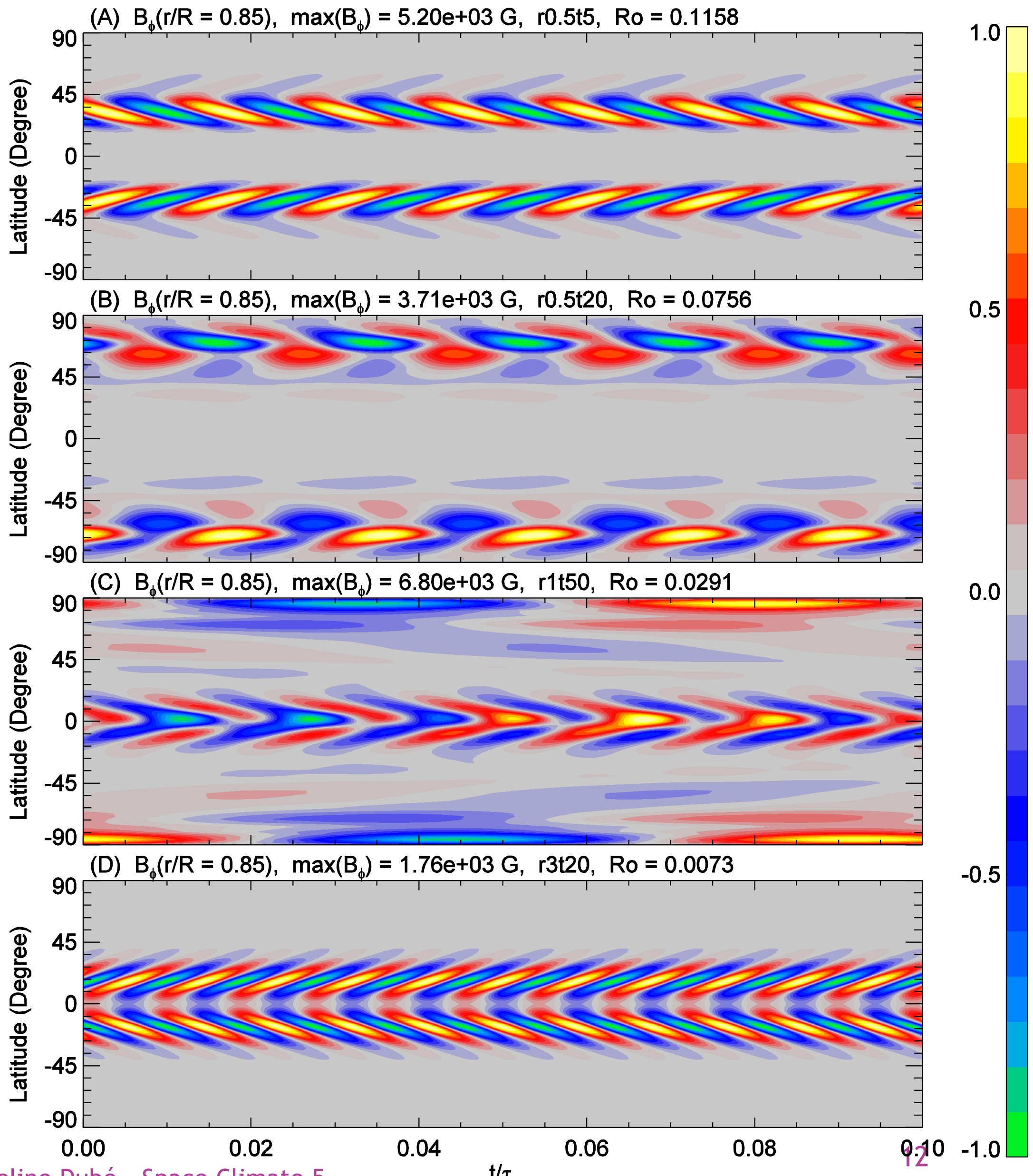
DYNAMO SOLUTIONS

$$\Omega_0/\Omega_\odot = 0{,}5$$
$$t_s = 5$$

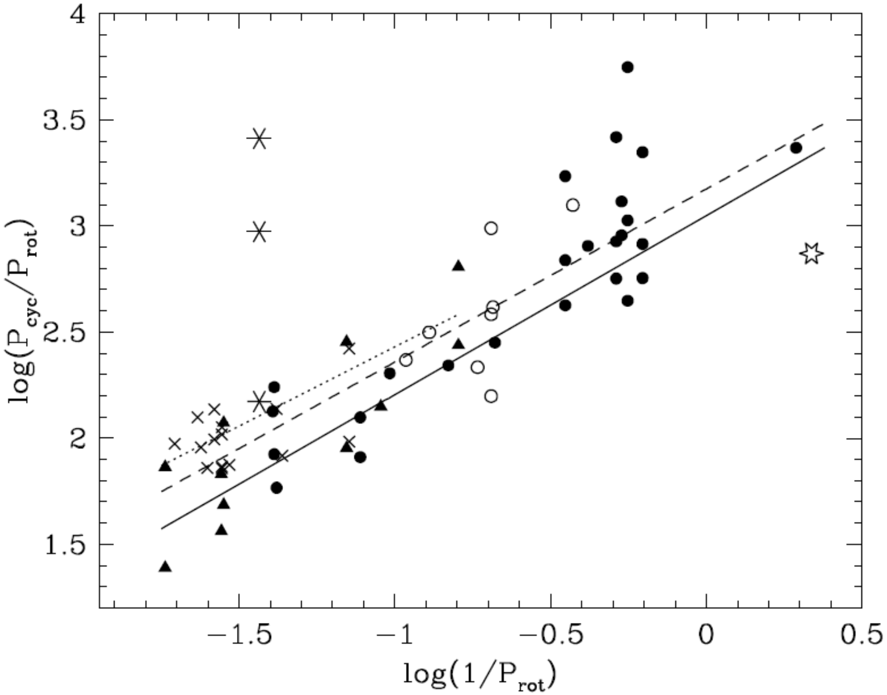
$$\Omega_0/\Omega_\odot = 0{,}5$$
$$t_s = 20$$

$$\Omega_0/\Omega_\odot = 1$$
$$t_s = 50$$

$$\Omega_0/\Omega_\odot = 3$$
$$t_s = 20$$



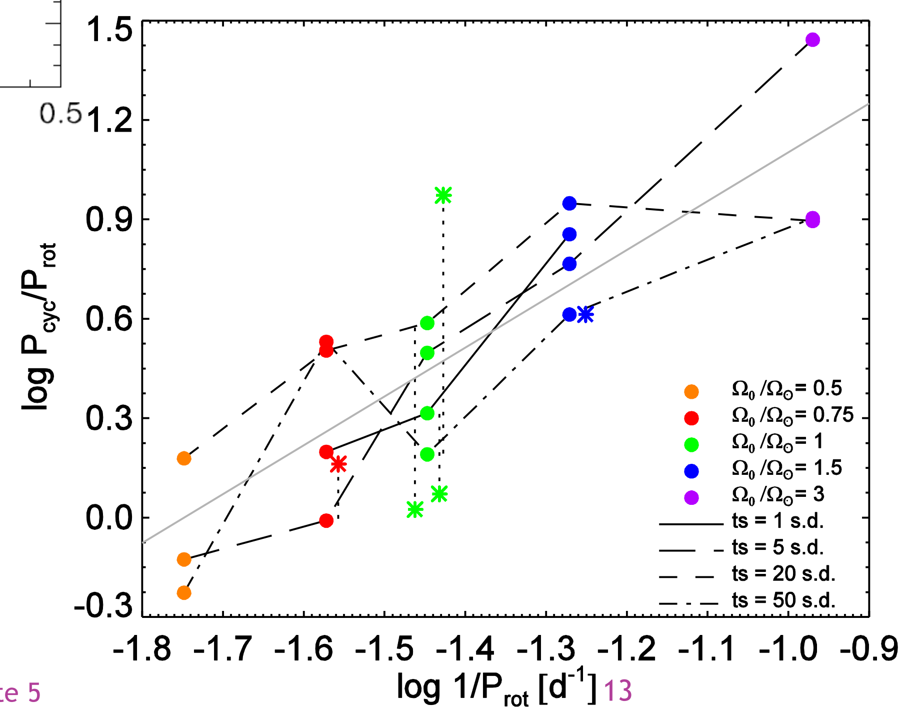
CYCLE RELATIONSHIPS



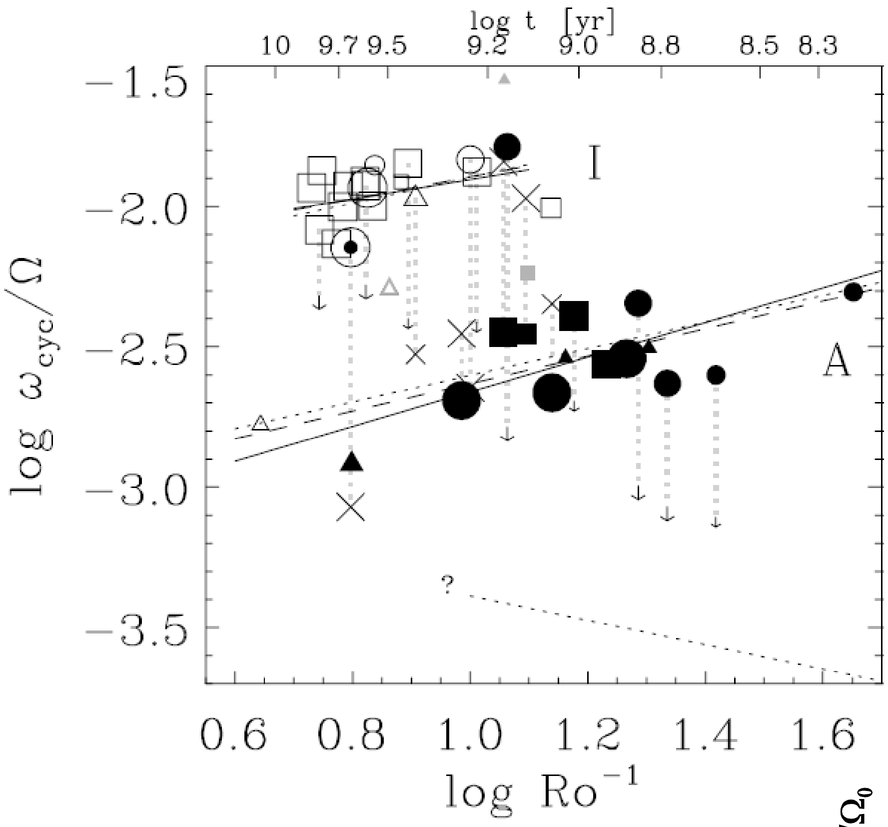
Oláh et al. 2009

- Trend reproduced: success
- Steeper slope:

  $1{,}47 \text{ vs. } 0{,}81 \pm 0{,}05$

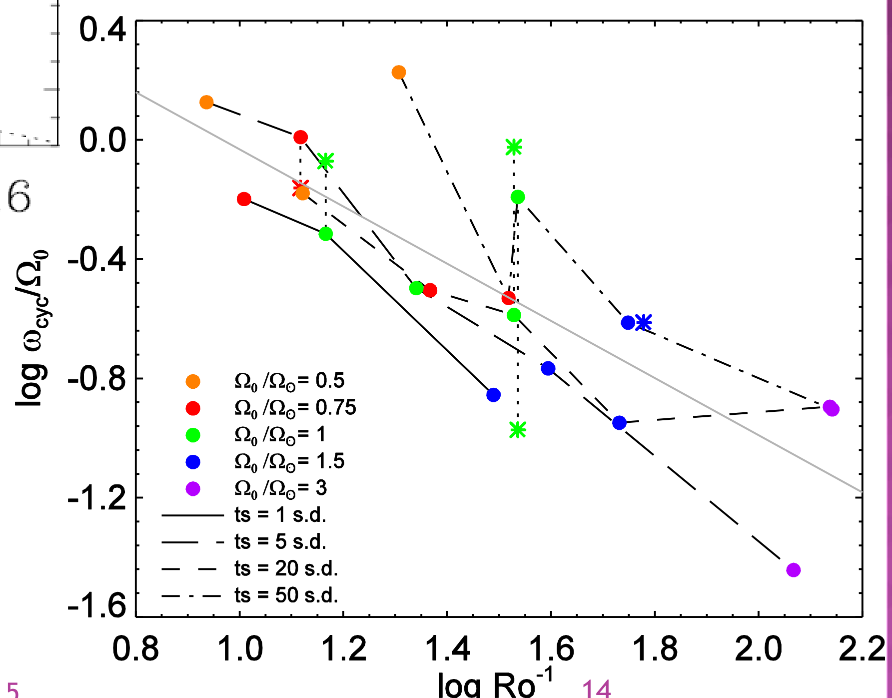


CYCLE RELATIONSHIPS

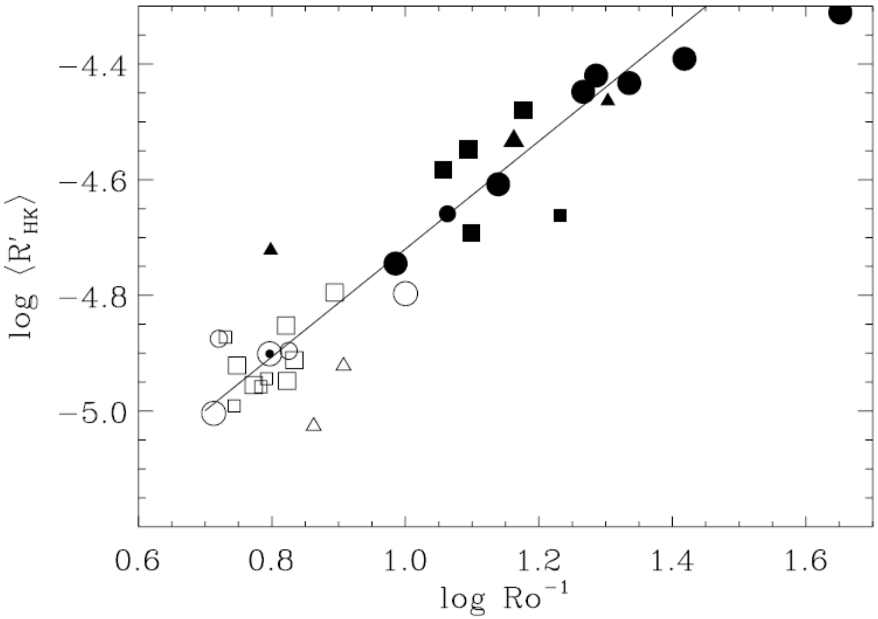


Saar & Brandenburg 1999

- Decreasing slope
- 2 branches not reproduced

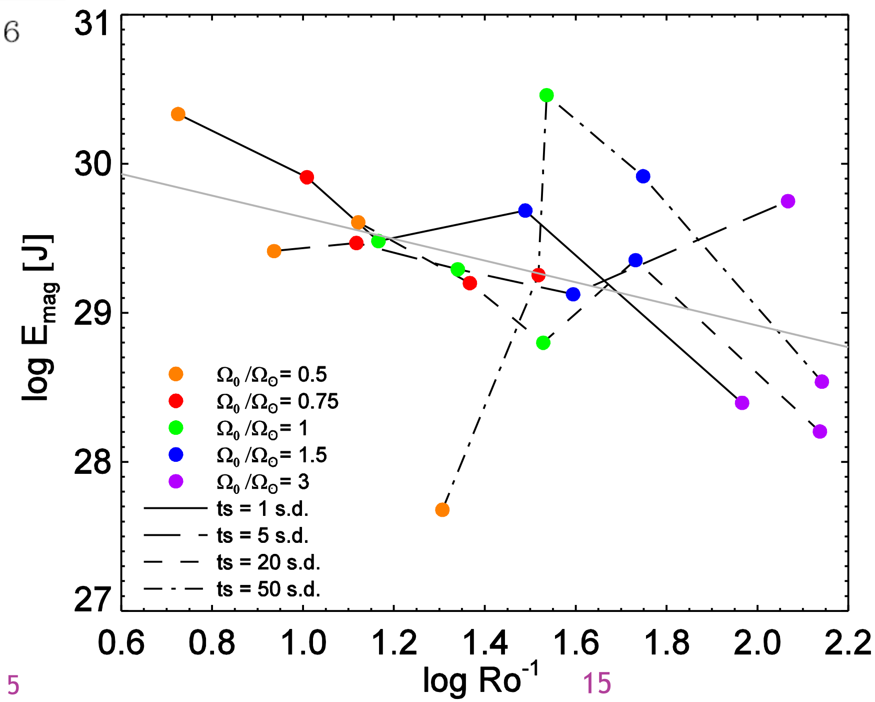


CYCLE RELATIONSHIPS



Saar & Brandenburg 1999

- Complete failure
- Likely due to the use of α-quenching as sole limiting amplitude mechanism



CAVEAT

- Solar internal structure model
- Implicit dependence of the Rossby number on the fluctuating part of the velocity's radial component
- Semi-empirical Rossby number used in the literature based on the variation of the luminosity and the convective flow in a 1:1 ratio
- α-quenching is the only saturation mechanism used while Racine et al. (2011), and others, suggest that magnetic backreaction on large-scale flows is the primary mechanism

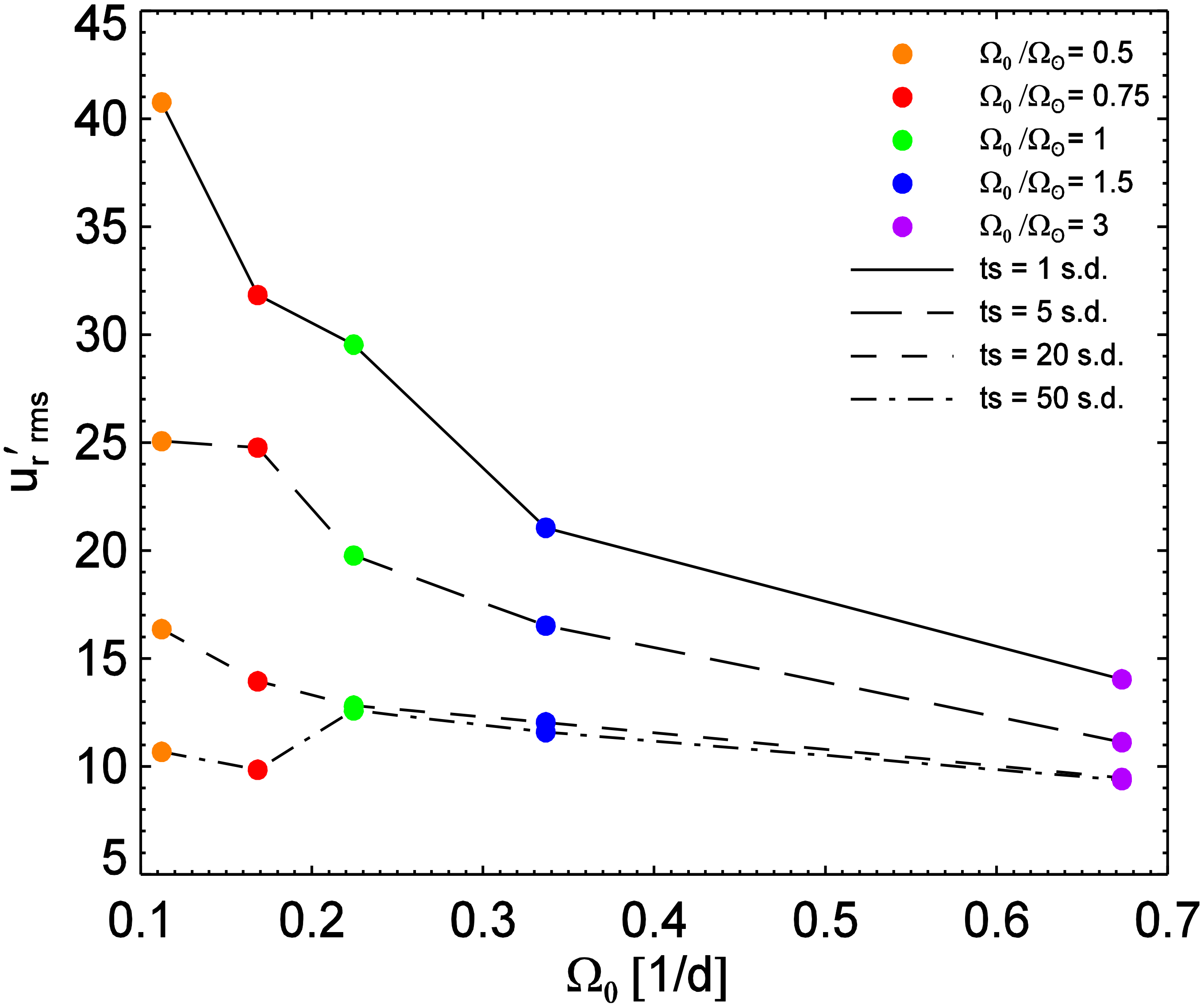
Results are model dependent

SUMMARY

- 20 different profiles of differential rotation and kinetic helicity using EULAG-HD
- $\alpha_{\phi\phi}$ was reconstructed using SOCA
- Input for a mean-field $\alpha\Omega$ dynamo model, we constructed the large scale magnetic field
- P_{cyc} vs P_{rot} relationship robust feature of $\alpha\Omega$ dynamo model; not so for Babcock-Leighton model
- Fail to reproduce other relationships
- α-quenching probably not principal mechanism for saturation
- Next step, EULAG-MHD simulations

THANK YOU

VARIATION OF $u'_{r\,rms}$



VARIATION OF f_c