REVERSALS OF GNEVYSHEV - OHL RULE

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The analysis of sunspots groups number in activity cycles since 1610 till present is performed. We use here the G_n index, which is defined as the average daily number of sunspots groups in cycle *n*. There is a high positive correlation between the parameter G_n in the current cycle and analogous parameter in the following cycle G_{n+1} both for pairs of even-odd cycles, and pairs of odd-even activity cycles. In cycles № 10÷21 for pairs of even-odd cycles the ratio of parameter G_n corresponds to the GO rule $G_{n+1}^{odd} / G_n^{even} > 1$. But in period ~1745-1850 odd cycles were less than the preceding even cycles $G_{n+1}^{odd} / G_n^{even} < 1$. The ratio of parameter $G_{n+1}^{odd} / G_n^{even}$ has a long-term variation within the range of 0.5 ÷ 1.5 with the period of about 21 activity cycles, and it proves the reversal of the GO rule.

1. INTRODUCTION

Empirical rule of Gnevyshev - Ohl (GO) is formulated for the pair of subsequent solar cycles. The article of Gnevyshev & Ohl 1948 contains the information about the analysis of annual average values of sunspot index R (Wolf number) for cycles № -4 ÷ 17. It was discovered that the sum of index $\sum R$ in even cycles 2n has a good positive correlation with the succeeding odd cycle 2n+1, while the correlation in pairs of odd cycle 2n-1 and even cycle 2n is weak. It allowed the authors to formulate the hypothesis that the 22nd cycle begins with an even cycle with respect to small magnitude. Then it is followed by an odd cycle, the magnitude of which is determined by the preceding cycle, and it indicates at close physical connection between them (Gnevyshev & Ohl 1948).

Nowadays there are several definitions of the GO rule: a) the amplitude of the even activity cycle is less than the amplitude of the following odd cycle; b) the sum of Wolf numbers in even cycle is less than the sum of the following odd cycle; c) the area under the curve of Wolf numbers in even cycle correlates with the area under the curve in odd cycles, at the same time even and the following odd cycle form a pair (Kopecky, 1950; Hathaway 2002; Nagovitsyn et al. 2009, Ogurtsov & Lindholm 2011). GO rule in its different definitions is justified for cycles № 10-21, but there are some violations for pairs 4-5, 8-9, 22-23 (Gnevyshev & Ohl 1948, Wilson 1988, Hathaway 2010).

Usually, to check GO rule one uses sequence of Wolf numbers, reconstructed by R. Wolf since 1749. However, this sequence has a significant noise in earlier observations and does not take other kinds of observation into consideration (Hoyt et al. 1994; Hoyt & Schatten 1998). Basing upon additional data, Hoyt & Schatten (1994) offered index of sunspots groups number, reconstructed in the period since 1610 till 1995. Index of sunspots groups number gives the best correlation ratio between the amplitudes of even and odd cycles in comparison with Wolf number (Hathaway et al. 2002). For checking GO rule, Mursula et al. (2001) offered to use index of sunspots groups number as $I_{GO}(k) = 1/132 \sum_{j=J(k)}^{J(k+1)-1} Rg$, where *Rg*- average monthly index of sunspots groups number, J(k)- the month of beginning of cycle *k*, invariable 1/132 is introduced for scaling of obtained index towards standard indexes of sunspots. The authors showed, that during the period since 1725 till 1782 even cycles have a larger amplitude, than the following odd cycles. For eliminate this discrepancy, they offered a hypothesis – one solar cycle was lost in the beginning of the Dalton minimum during 1790-s (Usoski et al. 2001; Usoskin et al. 2009). The index $I_{GO}(k)$ offered by the authors (Mursula et al. 2001) is analogous to the sum of the sunspots per cycle. But in case of a large gap in observation days there is a difficulty in calculating the sum of sunspots groups per cycle, as well as defining the amplitude of a cycle.

This paper offers to apply index of sunspots groups number, based not upon summing the number of sunspots groups per cycle, but upon calculating the average number of sunspots groups per day during one activity cycle.

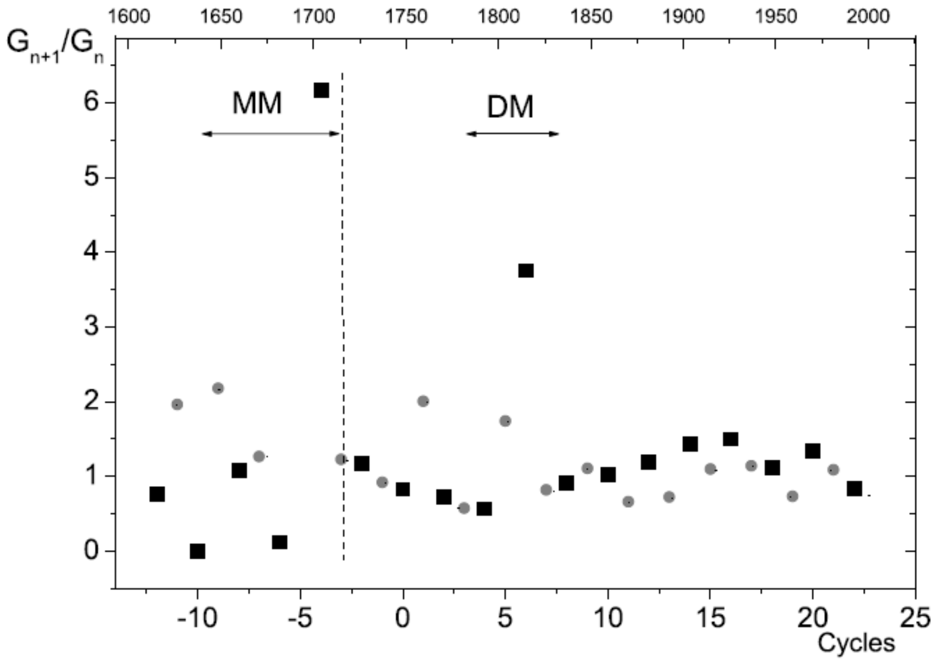

Figure 1. The ratio of average daily number of sunspots groups in neighboring cycles $G_{n+1/}/G_n$. Squares indicate at pairs G^{odd}_{n+1}/G^{even}_n, circles show G^{even}_{n+1}/G^{odd}_n. Positions of Maunder Minimum (MM) and Dalton Minimum (DM) are presented.

2. DATA AND ANALYSIS

To characterize the activity cycles we can apply daily average number of groups in a cycle:

$G_n = \sum_{T_n}^{T_{n+1}} g / N_d$, where g **is** amount of groups per current day, N_d **is** number of observation days in cycle n, T_n **is** the moment of the beginning of cycle n. Preliminary data (Hoyt et al. 1994) contain the information of different observatories and practicing astronomers about the number of sunspots groups per day (ftp://ftp.ngdc.noaa.gov/STP/SOLAR_DATA/SUNSPOT_NUMBERS/GROUP_SUNSPOT_NUMBERS/alldata.txt). Within this analysis, if there were series of observations, the one with the largest number of groups was chosen. The results were also checked by means of data with interpolate daily values. The data concerning the beginning of cycles T_n were also taken from the website NGDC. Figure 1 shows the values of parameter G_n in the period since 1610 till present. For cycles 23, 24 the values are performed during the calculation of number of the groups in accordance with the sunspots data base USAF/NOAA (http://solarscience.msfc.nasa.gov/greenwch.shtml).

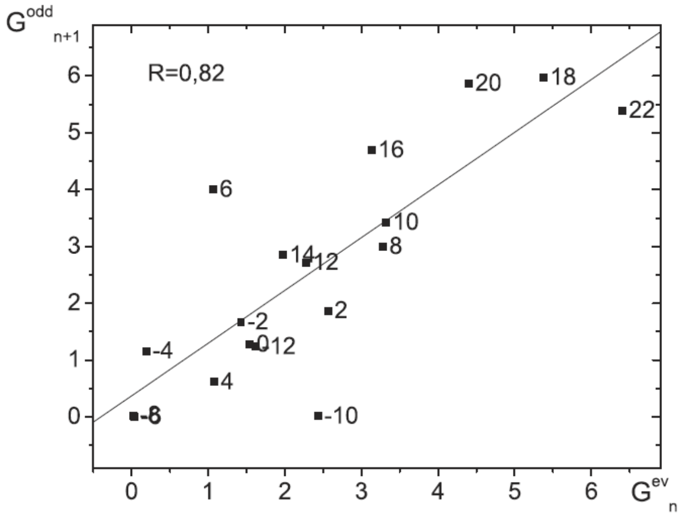

Figure 2. The average number of sunspot groups in the current cycle G_n^{even} compared with the number of sunspots groups in the following cycle G_{n+1}^{odd} for pairs of even-odd cycles.

The statistics of Wolf numbers exists since 1749 (cycle № 0). There is a positive correlation between index G_n and the amplitude of cycle in Wolf numbers with $W_n^{\max} = 45.0(12) + 19.0(3)\mathrm{G_n}; \quad \mathrm{r} = 0.8$.

Figure 1 shows time change in ratio of index of group number in the following activity cycle G_{n+1}, towards the preceding cycle G_n, for cycles № -12 ÷ 23. Pairs $G_{n+1}^{odd} / G_n^{even}$ and $G_{n+1}^{even} / G_n^{odd}$ are presented with different symbols. After the end of Maunder Minimum (1645-1715), starting from cycle № -2, for pairs of even and odd cycle, there is a ranking of values $G_{n+1}^{odd} / G_n^{even}$ in the form of a long-term modulation. The pair of cycle № 6-7 represents an exception, taking place during Dalton Minimum.

The average amount of sunspots groups in the following cycle G_{n+1} is linked with the number groups in the preceding cycle G_n. Figure 2 depicts function of G_{n+1} against G_n for pairs of even and the following odd cycles. The relation between G_n indexes in such pairs has a positive correlation $G_{n+1}^{odd} = 0.37(0.47) + 0.93(0.16)\, G_n^{even} \quad r = 0.82$, which corresponds to the standard GO rule. In addition to that, there is a high correlation for pairs of odd – even cycles (Fig. 3). The relation between indexes of such pairs comprised $G_{n+1}^{even} = 0.39(0.4) + 0.82(0.08)\, G_n^{odd}, \quad r = 0.91$.

Figure 1 shows that during the period № 10÷21 the average number of groups in odd cycles were higher than in preceding even cycles and the relation $G_{n+1}^{odd} / G_n^{even}$ corresponds to the standard formulations of GO rule, but is violated in pair of 22-23 cycles. Figure 4 represents the relation $G_{n+1}^{odd} / G_n^{even}$ in period after Maunder Minimum. All the pairs of cycles except from cycles 6-7 are within the range of values 0.5 ÷ 1.5. Starting from the cycle № -2 the relation $G_{n+1}^{odd} / G_n^{even}$ has a smooth envelope. As a comparison, the diagram shows the sinusoid with the period t=21 cycles and the amplitude a=0.45: $f(t) = 0.5 + 0.45 \cdot \sin(2\pi n / t)$. Standard deviation in ratio $G_{n+1}^{odd} / G_n^{even}$ from envelope curve amounted to $\sigma = 0.12$. χ^2-test gives the sinusoid a value of about 0.15 while for linear dependence it was about 0.69, which supports the proposed hypothesis.

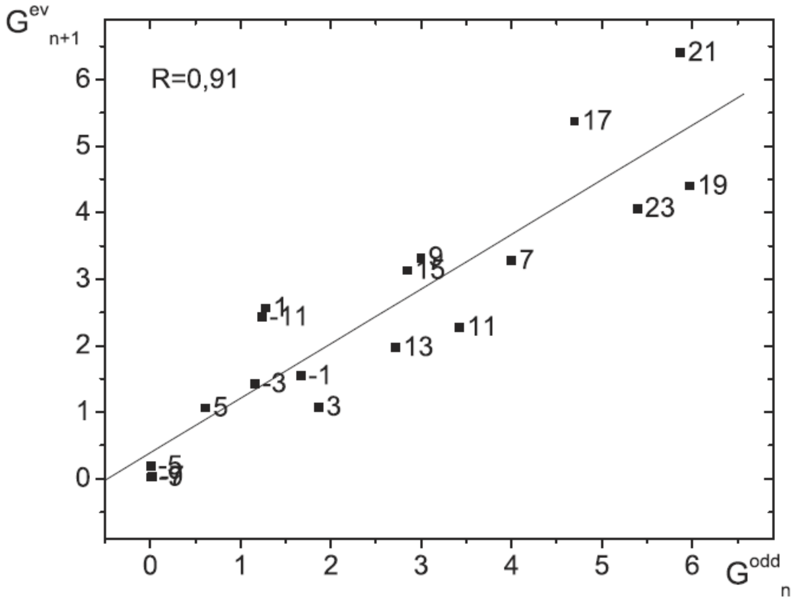

Figure 3. Just like in Fig. 2, but for pairs of odd G_n^{odd} and the following even activity cycles G_{n+1}^{even}.

3. DISCUSSION AND CONCLUSIONS

Research of the GO rule can give an important information about the nature of the solar periodicity, un particular, concerning the possible fossil solar magnetic field, with which one usually connects this effect (Bravo & Stewart 1995; Charbonneau 2009). Some authors think that the regularity when even cycles are less intensive, than the following odd ones, has a constant character (Usoskin et al. 2001; Nagovitsyn et al. 2009). However, the pair of 22-23 activity cycles obviously shows the violation of this rule (Hathaway 2010). Therefore, possibly, this rule had its reversals in previous centuries.

This paper uses G_n, the average daily number of sunspots groups in cycle *n* to test GO rule. We discovered that the ratio $G_{n+1}^{odd} / G_n^{even}$ in cycles № 10-21 in pairs of even-odd cycles was more than 1, while the pair 22-23 showed less than 1, and it corresponds to the results, obtained by means of Wolf numbers (Hathaway 2010). A strong correlation of G_n parameters was found in pairs of even-odd cycles for all the period under consideration, cycles № -12 ÷ 23 (r=0.82, Fig.2). These results correspond to the standard definitions of GO rule and its exception for pair 22-23, and it indicates at validity of using parameter G_n in testing GO rule. At the same time, a high positive correlation (r=0.91) was found in pairs of odd-even activity cycles (Fig. 3).

Applying index G_n allowed to single out a long-term envelop curve for values $G_{n+1}^{odd} / G_n^{even}$ after Maunder Minimum (Fig. 1). The changes are close to long-term variations with the period of about *t*~21 cycle (Fig. 4) or about 230 years. Only the pair of cycles № 6-7 is an exception, because it takes place in Dalton minimum. Some authors (Vitinsky et al. 1986; Mursula et al. 2001) came to the conclusion that apparition of the 22-year periodicity disappeared in the time when the level of solar activity changed quickly, for instance, during restoring of activity after Maunder Minimum, or closer to the Dalton minimum. Long-term cyclicity with the period about ~200-220 years had been found by means of reconstructing solar activity according to radioisotope data before (Suess 1980, Mordvinov & Kramynin, 2010; Abreu et al. 2012).

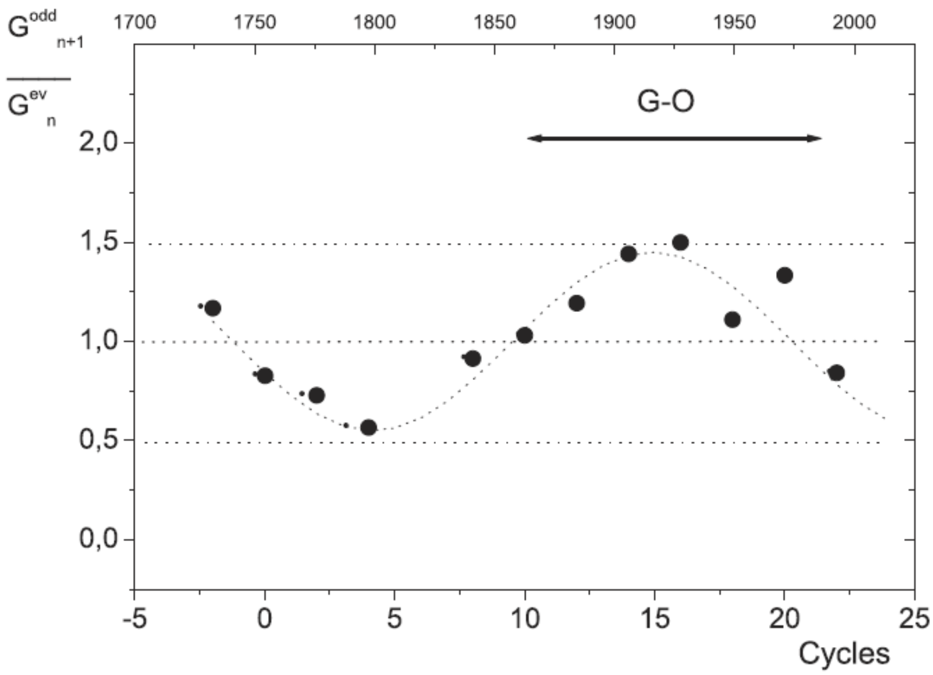

Figure 4. Ratio of average daily amount of sunspots groups in odd cycle to analogous value in the preceding in even cycle $G^{odd}_{n+1}/G^{even}_{n}$. An envelope line are drawn, and a line where this ratio is equal to 1.0, 0.5 and 1.5.

For period of ~1745-1850 the value of correlation in pars even-odd cycles $G^{odd}_{n+1}/G^{even}_{n}$ was less than 1. It proves that GO rule can reverse within long intervals, to be more exact – even cycles can be stronger than the following odd cycles. Duration of epochs, when $G^{odd}_{n+1}/G^{even}_{n} > 1$ and $G^{odd}_{n+1}/G^{even}_{n} < 1$ are approximately equal, and the reversal takes place during secular activity minimums (Fig. 4). It's possible to expect that the following activity cycles will develop within the reversed GO rule.

Presumably, the violation in 22-year cycles, when the ratio $G^{odd}_{n+1}/G^{even}_{n}$ becomes either more or less than 1 for a long time, has a periodic character, during which the Sun changes its cycle mode. As a rule, one can observe minimums of century variations of solar activity in the process.

To explain this, we can assume that in the long-term periods there is a permanent solar magnetic field which can also reverse, and it reverses the sequency of 22-year cycles. The reason of appearing of such a permanent field can be the so-called "magnetic memory" under the bottom generation zone (Tlatov 1996). This field appears during averaging out magnetic field of several subsequent cycles, having different direction of poloidal field, thus ensuring the relation G_{n+1}/G_{n} higher (lower) than 1 during long-term periods (Figure 1). The positive correlation between the preceding and the following cycle $G^{odd}_{n+1}/G^{even}_{n}$ and $G^{even}_{n+1}/G^{odd}_{n}$ (Figure 2, 3), as well as changes with long-term period, which depicts Figure 1, count in favor of this hypothesis, as well as long-term changes, visible in Figures 1, 4.

The average number G_n of sunspots groups in cycle *n*, unlike the total number R_n of sunspots, shows high correlation in ratio G_{n+1}/G_{n} both in pairs of even-odd and odd-even cycles. In this way applying index G_n differs from the standard GO rule. It is possibly, connected with changes of number of spots in sunspots groups in activity cycles. At the same time only pairs of even and odd cycles show long-term changes for sure (Fig.4). Thus, there are differences in the 22-year magnetic cycle for pairs of even-odd and for pairs of odd-even cycles, and there are differences in index G_n.

The violation of GO rule in 22-23 activity cycles can be a sign of changing in the character of periodicity period and long-term reversal of GO rule in the following activity cycles. GO rule, established and correct for cycles 10-21, is a part of long-period inequality of solar activity.

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