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Long term variabilty of the solar dynamo

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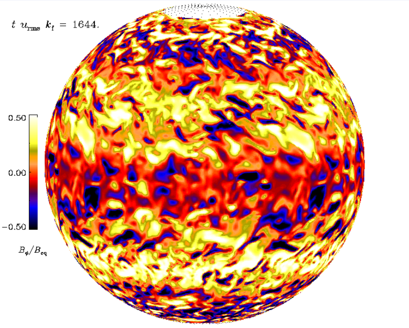
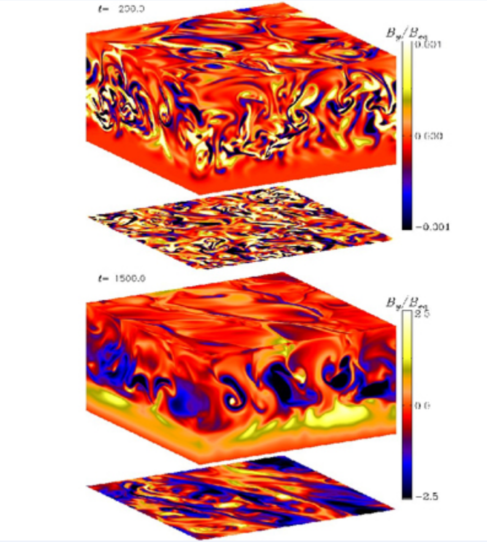
”millenium” solar-like dynamo





Global spherical convection dynamo e.g. (Käpylä et al. 2013, Käpylä et al. 2015)

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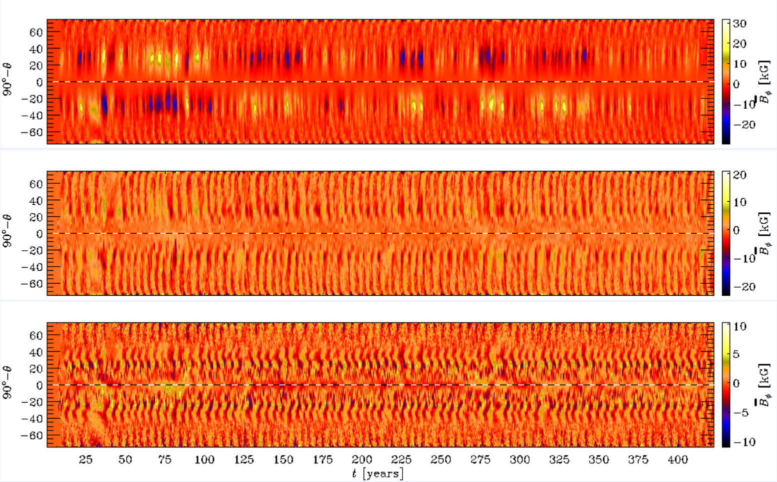

Figure: B_ϕ averaged azimuthally as function of latitude over time - layers near the base, middle and surface of the convection zone. Time derived by $5\Omega_\odot / R_\odot$, for a solar size star rotating 5x solar rate

model MHD equations

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$$\boldsymbol{F}^{\rm rad} = -K\boldsymbol{\nabla} T \quad \text{and} \quad \boldsymbol{F}^{\rm SGS} = -\chi_{\rm SGS}\rho T\boldsymbol{\nabla} s \qquad (5)$$

are heat fluxes, radiative and SGS (sub grid scale - numerical stability)

model MHD equations - symbols

$\boldsymbol{A}$	magnetic vector potential
$\boldsymbol{U}$	velocity
$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$	magnetic field
$\boldsymbol{J} = \mu_0^{-1} \boldsymbol{\nabla} \times \boldsymbol{B}$	current density
μ_0	vacuum permeability
$D/Dt = \partial/\partial t + \boldsymbol{u} \cdot \boldsymbol{\nabla}$	material derivative
$\mathbf{S}$	rate of strain tensor
ρ	density
ν	kinematic viscosity
η	magnetic diffusivity
K	radiative heat conductivity
χ_{SGS}	turbulent heat conductivity (unresolved convective transport of heat)
s	specific entropy
T	temperature
p	pressure

Ideal gas law: $p = (c_{\mathrm{P}} - c_{\mathrm{V}})\rho T$, where adiabatic index $\gamma = c_{\mathrm{P}}/c_{\mathrm{V}} = 5/3$.

long term variation of magnetic cycle

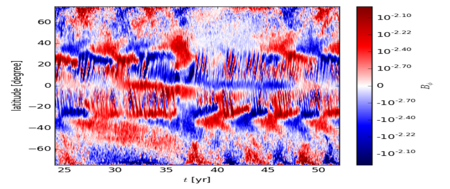
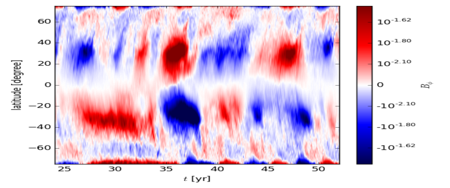


Figure: $\langle B_\phi \rangle_\phi$ near surface of the convection zone during grand minima south then north.



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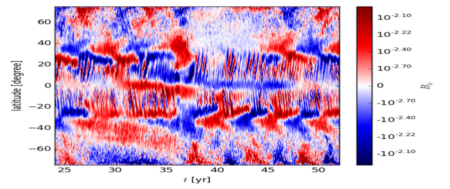

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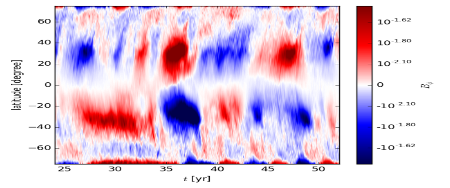

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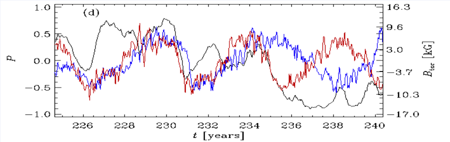

Figure: Parity (black) and $\langle B_\phi \rangle_\phi$ near surface (N:blue, S:red) at $\pm 25^\circ$ latitude, during high state of base toroidal mode

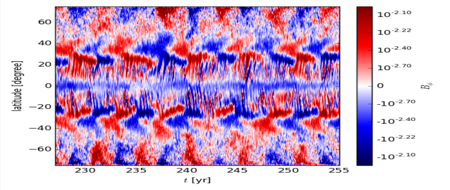

Figure: $\langle B_\phi \rangle_\phi$ near the surface of the convection zone during switch from N-S symmetry to asysmmetry.

mean field concept

(Schrinner et al. 2007) expressed the induction equation in terms of the mean field (e.g. azimuthal average)
such that $\boldsymbol{B} = \overline{\boldsymbol{B}} + \boldsymbol{b}$ and $\boldsymbol{U} = \overline{\boldsymbol{U}} + \boldsymbol{u}$
Taking the curl of (1)

$$\frac{\partial}{\partial t}(\overline{\boldsymbol{B}} + \boldsymbol{b}) = \nabla \times (\overline{\boldsymbol{U}} + \boldsymbol{u}) \times (\overline{\boldsymbol{B}} + \boldsymbol{b}) + \eta \nabla^2 (\overline{\boldsymbol{B}} + \boldsymbol{b}), \quad (6)$$

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where the electromotive force (EMF) $\mathcal{E} = \overline{\boldsymbol{u} \times \boldsymbol{b}}$

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where $\boldsymbol{G} = \boldsymbol{u} \times \boldsymbol{b} - \overline{\boldsymbol{u} \times \boldsymbol{b}}$

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$\mathcal{E}$ is a functional of $\boldsymbol{u}$, $\overline{\boldsymbol{U}}$ and $\overline{\boldsymbol{B}}$, i.e. linear in $\overline{\boldsymbol{B}}$

Let us assume that $\boldsymbol{b}$ vanishes for $\bar{\boldsymbol{B}} \to 0$

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Further assume only weakly in space and time
In general, if no higher than first order spatial derivatives and no time derivatives of $\overline{\boldsymbol{B}}$ are taken into account then in general

$$\mathcal{E} = \mathbf{a}\overline{\boldsymbol{B}} + \mathbf{b}\nabla\overline{\boldsymbol{B}}$$

with second rank $\mathbf{a}$ and third rank $\mathbf{b}$ tensors
i.e. 36 independent coefficients

mean field concept

in curvelinear coordinates can be expressed

$$\mathcal{E} = \alpha\overline{\boldsymbol{B}} + \gamma \times \overline{\boldsymbol{B}} - \beta \cdot (\nabla \times \overline{\boldsymbol{B}}) - \delta \times (\nabla \times \overline{\boldsymbol{B}}) - \kappa \cdot (\nabla\overline{\boldsymbol{B}})^{\mathrm{sym}} \quad (9)$$

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Coefficients, vectors γ and δ, second rank tensors α and β, and third rank tensor κ represent a decomposiiton of the velocity field and can be related to physical processes, helping us to understand the structure of the dynamo.

Assuming axisymmetry in spherical geometry, then reduce to 27 independent coefficients

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How might we determine these coefficients?

Assume Eq.(8) for $\boldsymbol{b}$ pertains to a steady test field $\overline{\boldsymbol{B}}_T$

$$\frac{\partial \boldsymbol{b}}{\partial t} - \nabla \times (\overline{\boldsymbol{U}} \times \boldsymbol{b}) - \nabla \times \boldsymbol{G} - \eta \nabla^2 \boldsymbol{b} = \nabla \times (\boldsymbol{u} \times \overline{\boldsymbol{B}}_T) \qquad (10)$$

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For any given test field $\overline{\boldsymbol{B}}_T^{(i)}$ the EMF, depending only on $\overline{\boldsymbol{U}}$ and $\boldsymbol{u}$, can be expressed

$$\mathcal{E}^{(i)} = \tilde{a}_{jk} \overline{\boldsymbol{B}}_{T_k}^{(i)} + \tilde{b}_{jkr} \frac{\partial \overline{\boldsymbol{B}}_{T_k}^{(i)}}{\partial r} + \tilde{b}_{jk\theta} \frac{1}{r} \frac{\partial \overline{\boldsymbol{B}}_{T_k}^{(i)}}{\partial \theta} \qquad (11)$$

Applying these to 9 linearly independent axisymmetric test fields we can solve simultaneously for the 27 independent coefficients

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How plausible is the mean field model?

verification of mean field model

So, assuming we have the coefficients and can analyse what role the various processes play in the dynamo.

Is it meaningful?

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How does the mean field dynamo compare with the full solution?

magnetoconvection-*B*

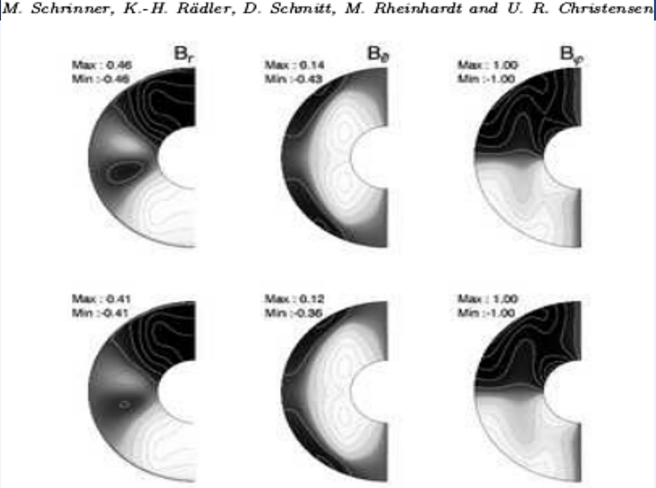

Figure: (Schrinner et al. 2007) magnetoconvection: azimuthally averaged magnetic field components resulting from DNS (upper), mean-field calculations derived from test field (lower). $[(\rho\mu_0\eta\Omega)^{1/2}]$

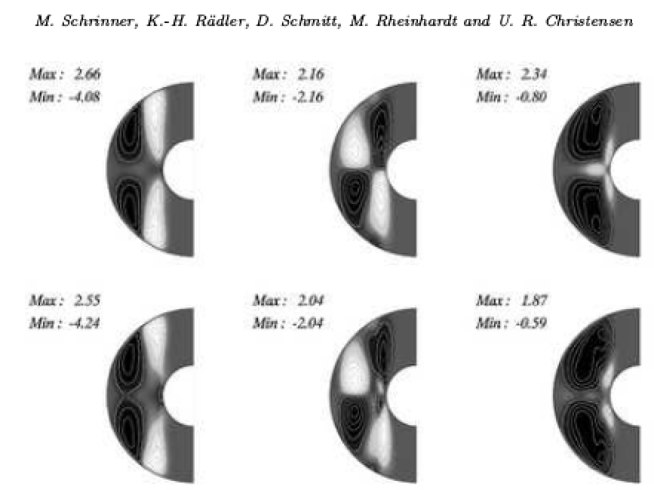

Figure: (Schrinner et al. 2007) electromotive forces in the magnetoconvection (top) $\mathcal{E}_r^{\mathrm{MHD}}$, $\mathcal{E}_\theta^{\mathrm{MHD}}$, $\mathcal{E}_\phi^{\mathrm{MHD}}$, and (bottom) $\mathcal{E}_r^{\mathrm{MF}}$, $\mathcal{E}_\theta^{\mathrm{MF}}$, $\mathcal{E}_\phi^{\mathrm{MF}}$.
$[(\eta/D)(\rho\mu_0\eta\Omega)^{1/2}]$

test field application to millenium data

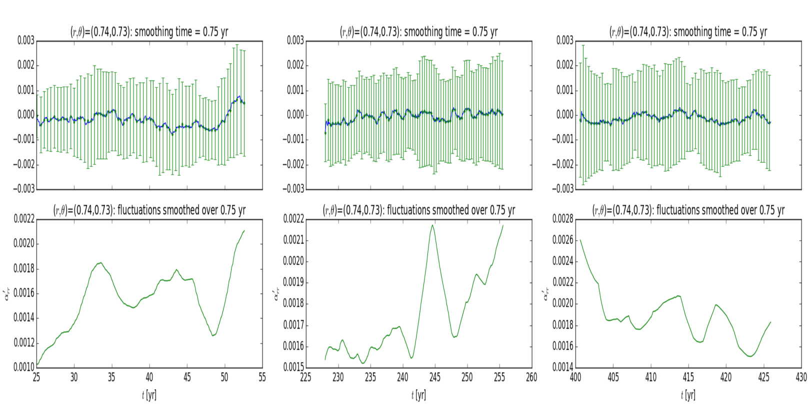

Figure: Upper: $\alpha_{rr}(0.74R_\odot, 0.73\mathrm{rad})$, box-car averaged over 9 month intervals (blue), standard deviation over interval (error bars). Lower: Perturbations $\alpha'_{rr_{\mathrm{rms}}}$ box-car averaged over 9 months

time averaged α-tensor steady cyclic

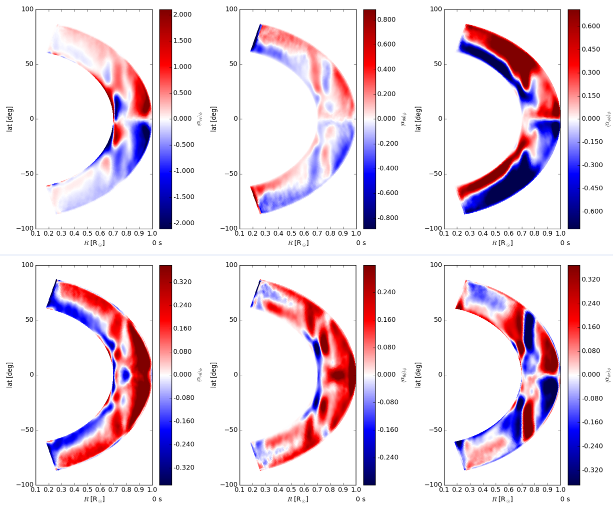

Figure: Time and azimuthally averaged α-tensor for the period corresponding to steady cyclic epoch

time evolution α-tensor - base

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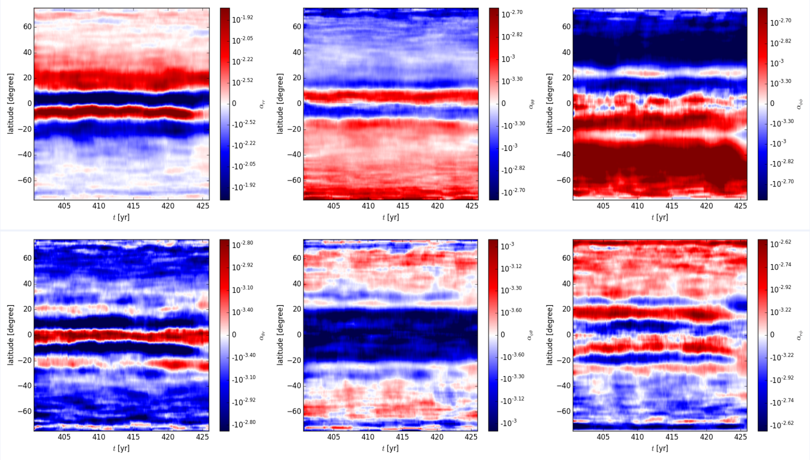

Figure: Time evolution of azimuthally averaged α-tensor spanning steady cyclic epoch near the base of the convection zone

time evolution α-tensor - surface

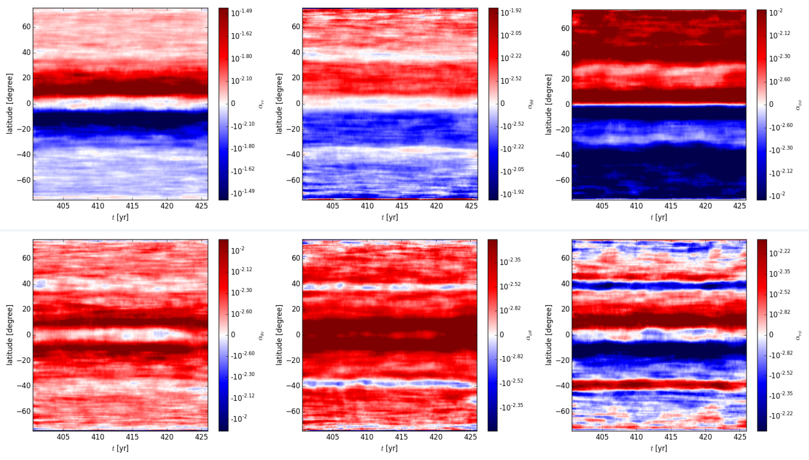

Figure: Time evolution of azimuthally averaged α-tensor spanning steady cyclic epoch near the surface of the convection zone

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  fluctuations about 10x mean values - but follow mean magnitude?
- mean field model may work with simple random fluctuations?

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